Tachyonic thermal excitations and causality

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October 11, 2011

Abstract

We consider an ideal Fermi gas of tachyonic thermal excitations as a continuous medium and establish when it satisfies the causality condition. At high temperature the sound speed is always subluminal $c_s < 1$, but there is no stable form of tachyon matter below the critical temperature $T < T_c = 0.23m$ that depends on the tachyon mass m. The pressure P and energy density E cannot be arbitrary small, but P can exceed E, and $P = 2.36E$ when $T \to T_c$.

1   Introduction

Tachyons, first introduced for the description of superluminal motion [1, 2], are commonly known in the field theory as instabilities with energy spectrum

$$\varepsilon_k = \sqrt{k^2 - m^2} \qquad k > m \qquad (1)$$

where m is the tachyon mass and relativistic units $c = \hbar = 1$ are used. The concept of tachyon fields plays significant role in the modern research, and tachyons are considered as candidates for the dark matter and dark energy [3, 4], they often appear in brane theories [5] and cosmological models [6, 7].

A system of many tachyons can be studied in the frames of statistical mechanics [8, 9], and thermodynamical functions of ideal tachyon Fermi and Bose gases are calculated [10, 11]. We have recently studied the equation of state (EOS) and acoustic properties of the cold tachyon Fermi gas, which cannot be stable at arbitrary low density [12].

In the present paper we consider fermionic thermal excitations that can appear in material medium at finite temperature. Thermal excitations with the energy spectrum

$$\varepsilon_k = \sqrt{k^2 + m^2} \tag{2}$$

and finite energy gap $m = \Delta$ are well known in the theory of superconductivity [13, 14]. Excitations with tachyonic energy spectrum (1) are something exotic. We consider an ideal gas of thermal excitations as a continuous medium, calculate the sound speed

$$c_s^2 = \frac{dP}{dE} \tag{3}$$

and check the causality condition

$$c_s \leq 1 \tag{4}$$

The main task of the study: to clarify either (4) is satisfied at arbitrary temperature T or only within narrow range of thermodynamical parameters.

2   Tachyon Fermi gas

Consider a system of free fermions with the energy spectrum ε_k. Its energy density E, pressure P and particle number density n are defined by formuals [15]:

$$E = \frac{\gamma}{2\pi^2} \int \varepsilon_k \, f_k \, k^2 dk \tag{5}$$

$$P = \frac{\gamma}{6\pi^2} \int \frac{\partial \varepsilon_k}{\partial k} f_k \, k^3 dk \tag{6}$$

$$n = \frac{\gamma}{2\pi^2} \int f_k \, k^2 dk \tag{7}$$

where γ is the degeneracy factor, and

$$f_k = \frac{1}{\exp\left(\varepsilon_k - \mu\right) + 1} \tag{8}$$

is the distribution function. If we consider thermal excitations in some material medium, their number will not be conserved, and we put the chemical potential equal to zero $\mu = 0$.

For massive subluminal excitations with the energy spectrum (2) the thermodynamical functions (5)-(7) are determined by formulas

$$E = \frac{\gamma T^4}{2\pi^2} \int_{\beta}^{\infty} \frac{\sqrt{x^2 - \beta^2}\, x^2 dx}{\exp x + 1} \tag{9}$$

$$P = \frac{\gamma T^4}{6\pi^2} \int_{\beta}^{\infty} \frac{\sqrt{(x^2 - \beta^2)^3}\, dx}{\exp x + 1} \tag{10}$$

$$n = \frac{\gamma T^3}{2\pi^2} \int_{\beta}^{\infty} \frac{\sqrt{x^2 - \beta^2}\, x dx}{\exp x + 1} \tag{11}$$

where we have introduced dimensionless variables

$$x = \frac{\varepsilon_k}{T} \qquad \beta = \frac{m}{T} \tag{12}$$

For excitations with the tachyonic energy spectrum (1) the formulas (5)-(7) are presented so

$$E = \frac{\gamma T^4}{2\pi^2} \int_{0}^{\infty} \frac{\sqrt{x^2 + \beta^2}\, x^2 dx}{\exp x + 1} \tag{13}$$

$$P = \frac{\gamma T^4}{6\pi^2} \int_{0}^{\infty} \frac{\sqrt{(x^2 + \beta^2)^3}\, dx}{\exp x + 1} \tag{14}$$

$$n = \frac{\gamma T^3}{2\pi^2} \int_{0}^{\infty} \frac{\sqrt{x^2 + \beta^2}\, x dx}{\exp x + 1} \tag{15}$$

The thermodynamical functions of massless fermionic excitations will be given by the same formulas (13)-(15) or (9)-(11) when we put $\beta = 0$ in all integrals.

The particle number density of tachyons and massive subluminal particles is given in Fig. 1. The ratio P/E vs $\beta = m/T$ is given in Fig. 2. The equation of state of ultrarelativistic gas

$$P = \frac{E}{3} \tag{16}$$

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is achieved in the high-temperature limit $\beta \to 0$.

Substituting (13) and (14) in (3) we find the sound speed

$$c_s^2 = \frac{dP}{dT}\left(\frac{dE}{dT}\right)^{-1} \tag{17}$$

that is

$$c_s^2 = \frac{\int\limits_0^\infty \beta^2\sqrt{x^2+\beta^2} - \frac{4}{3}\sqrt{(x^2+\beta^2)^3}\,\frac{dx}{e^x+1}}{\int_0^\infty\left(\frac{\beta^2}{\sqrt{x^2+\beta^2}} - 4\sqrt{x^2+\beta^2}\right)x^2\,\frac{dx}{e^x+1}} \tag{18}$$

The sound speed in the gas of massive subluminal fermions is calculated by formula (17) with the energy density and pressure taken from (9) and (10). The behavior of the sound speed is explained in Fig. 3. At high temperature ($\beta \ll 1$) it tends to the limit $c_s = 1/\sqrt{3}$ corresponding to the sound speed in ultrarelativistic ideal gas.

The heat capacity is determined by formula

$$C_V = T\frac{\partial S}{\partial T} = -\beta\frac{\partial S}{\partial \beta} \tag{19}$$

where the entropy density is

$$S = \frac{\partial P}{\partial T} = -\frac{\beta^2}{m}\frac{\partial P}{\partial \beta} \tag{20}$$

Substituting (14) in (19) and in (20), we have

$$C_V = \frac{\gamma T^3}{2\pi^2}\int\limits_0^\infty\left(4\sqrt{(x^2+\beta^2)^3} - 5\beta^2\sqrt{x^2+\beta^2} + \frac{\beta^4}{\sqrt{x^2+\beta^2}}\right)\frac{dx}{e^x+1} \tag{21}$$

It is always larger than the heat capacity of massless Fermi gas

$$C_0 = \frac{7\gamma\pi^2 T^3}{60} \tag{22}$$

and the latter is larger than the heat capacity of massive subluminal fermions

$$C_V = \frac{\gamma T^3}{2\pi^2}\int\limits_\beta^\infty\left(4\sqrt{(x^2-\beta^2)^3} + 5\beta^2\sqrt{x^2-\beta^2} + \frac{\beta^4}{\sqrt{x^2-\beta^2}}\right)\frac{dx}{e^x+1} \tag{23}$$

The heat capacity C_V vs β is given in Fig. 4. At high temperature ($\beta \ll 1$) they all tend to (22), while at low temperature ($\beta \gg 1$) the discrepancy between tachyonic and subluminal excitations is much more evident.

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3 Discussion

An ideal gas of tachyonic thermal excitations reveals peculiar properties. Its particle number density (18) is always higher than the particle number density massless excitations

$$n_0 = \frac{\gamma \zeta(3) T^3}{\pi^2} \tag{24}$$

or the particle number density of massive subluminal excitations at the same temperature (18), see Fig. 1).

The sound speed (18) in an ideal gas of tachyonic excitations (Fig. 2) is superluminal when

$$\beta > \beta_c \cong 4.36 \tag{25}$$

i.e. when

$$T < T_c \cong 0.23m \tag{26}$$

Thus, an ideal Fermi gas of tachyonic thermal excitations satisfies the causality (4) and can form continuous medium when its temperature is higher the critical value T_c (26).

While the equation of state of massive subluminal excitations is "softer" than the EOS of photon gas

$$P < \frac{E}{3} \tag{27}$$

the EOS of tachyonic excitations is always relatively "stiff"

$$P > \frac{E}{3} \tag{28}$$

and when

$$\beta > \beta_1 \cong 2.69m \tag{29}$$

it becomes "hyperstiff"

$$P > E \tag{30}$$

see Fig. 2. At the critical temperature T_c (26) the tachyonic gas attains minimum possible value of the energy density and pressure

$$E_c = 1.40 \times 10^{-3} \gamma m^4 \tag{31}$$

$$P_c = 3.32 \times 10^{-3} \gamma m^4 \tag{32}$$

while
$$P_c \cong 2.36 E_c \tag{33}$$

For example, for the nucleon mass $m = m_p = 939 \, \text{MeV}$ formulas (31)-(32) yield the following estimation

$$E_c = 282 \, \text{MeV} \cdot \text{fm}^{-3} \tag{34}$$

$$P_c = 670 \, \text{MeV} \cdot \text{fm}^{-3} \tag{35}$$

that is of the same order as the nuclear energy $m_p n_{nm} = 158 \, \text{MeV} \cdot \text{fm}^{-3}$ at the saturation density $n_{nm} = 0.17 \, \text{fm}^{-3}$.

The heat capacity of tachyons is higher than the heat capacity of massless and massive subluminal fermions (see Fig. 4) attaining its maximum value

$$C_V \cong 1.70 \gamma T_c^3 = 1.48 C_0 \tag{36}$$

at $T \to T_c$ and exceeding almost 4 times the heat capacity of massive subluminal fermions at the same temperature.

The main conclusion of the present study: tachyonic thermal excitations result in the breaking of the causality (4) when the temperature is below the critical value $T < T_c$ (26), and their EOS near this point is "hyperstiff" (30). This may give new ideas for further research of material medium with tachyonic excitations, where the latter may cause instability of the whole composite system.

The authors are grateful to Erwin Schmidt for helpful conversations.

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Figure 1: The particle number density of ideal gases of thermal excitations in the unit of n_0 (24) vs temperature variable $\beta = m/T$. Solid line: tachyonic fermions, dotted line: massive subluminal fermions.
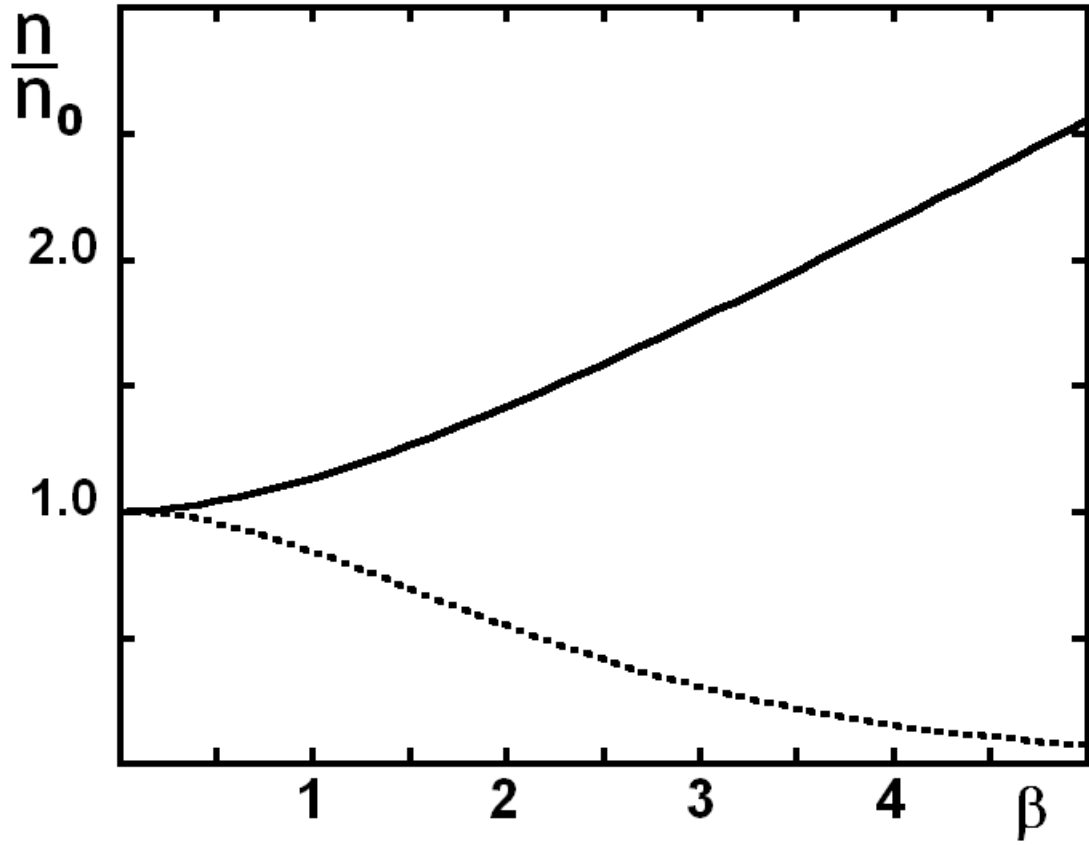
Figure 2: The ratio of pressure to the energy density P/E vs $\beta = m/T$. Solid line: tachyonic fermions, dotted line: massive subluminal fermions.
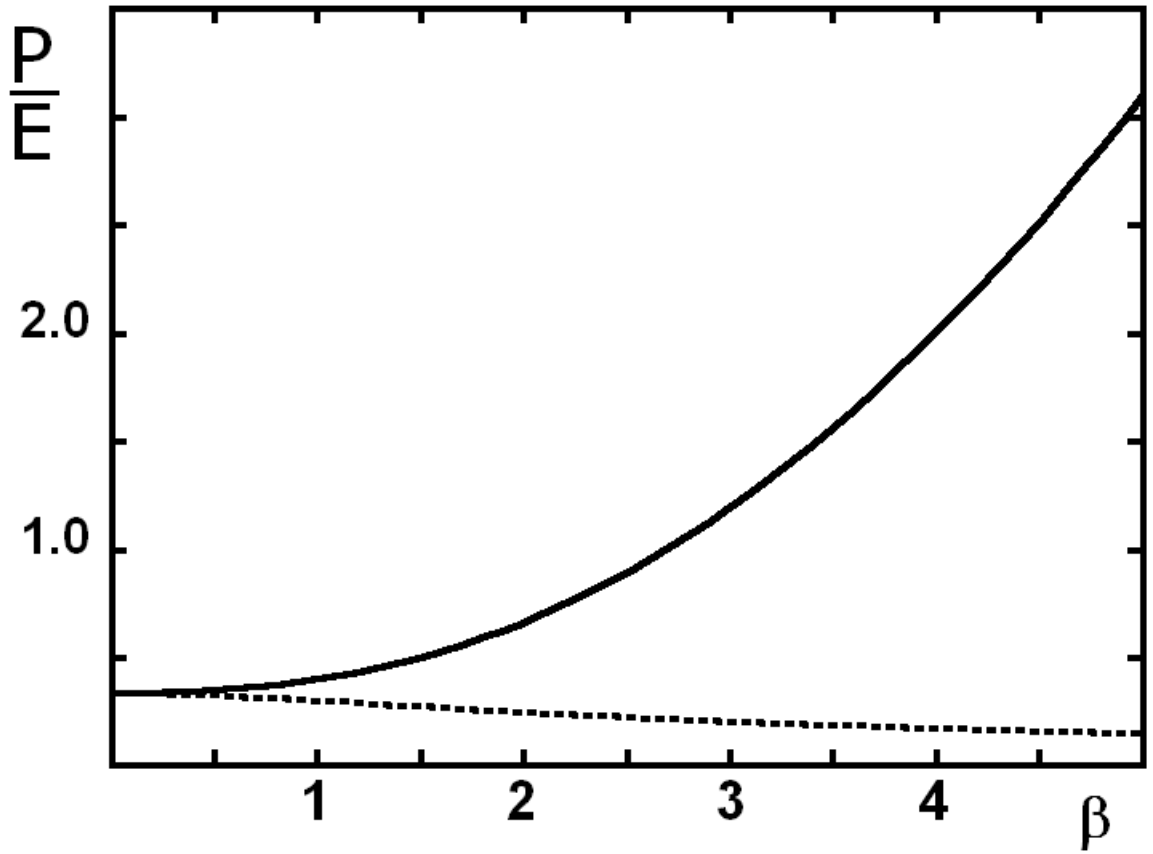
Figure 3: Sound speed c_s^2 vs $\beta = m/T$. Solid line: tachyonic fermions, dotted line: massive subluminal fermions
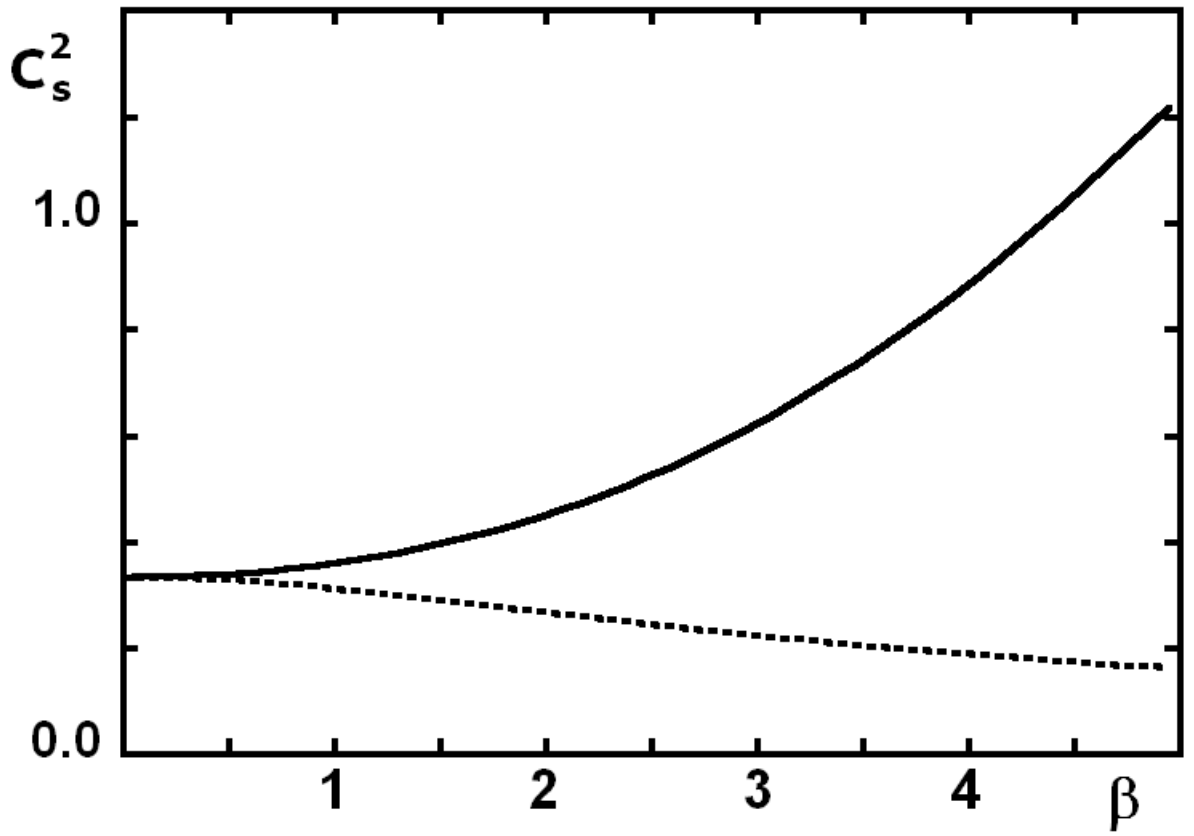
Figure 4: Heat capacity C_V in the unit of C_0 (22) vs $\beta = m/T$. Solid line: tachyonic fermions, dotted line: massive subluminal fermions